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STABILITY ANALYSIS OF ADAPTIVELY CONTROLLED NOT-NECESSARILY MINIMUM PHASE SYSTEMS WITH DISTURBANCES*

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Résumé :

Nous présentons une approche pour étudier la stabilité de systèmes commandés adaptativement et sujets à des perturbations bornées de statistiques inconnues. Une démonstration plus courte de certains résultats établis pour les systèmes à minimum de phase est donnée. Des résultats originaux sont établis pour les systèmes à non minimum de phase. Dans tous les cas, la bornitude de tous les signaux est obtenue moyennant une condition peu restrictive. Des exemples de schémas adaptatifs pouvant être traités par cette approche sont présentés.

Abstract :

An approach for studying the stability of adaptively controlled not-necessarily minimum phase systems subject to bounded disturbances is presented. A shorter proof of some results established for minimum phase systems is given. New results are established for non-minimum phase systems. In each case, the uniform boundedness of all signals is proven under a condition little restrictive in practice. Examples of adaptive schemes that can be treated with this approach are given.

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INTRODUCTION

Several recent studies analyze the stability of adaptively controlled systems. Many of these studies are more specifically concerned with linear systems without disturbances [1-6]. The results obtained in the disturbance-free case are not however entirely satisfactory in that they do not hold for processes with disturbances. Moreover, it has been shown [11] that the introduction of disturbances, even very small, in the system could deteriorate the efficiency of the control to the point of leading to the instability of the closed-loop system. The analysis of the stability of the adaptively controlled stochastic systems therefore seems to be of interest since, while taking into account the disturbances acting on the system, it allows in certain cases the establishment of the convergence of the adaptive control to an optimal control [7-10]. Yet, until now, this analysis lies upon the use of Martingale theorems that only apply if the disturbance acting on the system verifies the statistical properties of a moving average of independent zero mean random variables. In practice such conditions are hardly ever completely satisfied and this type of analysis can be reproached, as also in the disturbance-free case, for lacking robustness. Moreover, using a Martingale theorem in the stability analysis seems to forbid the use of non-decreasing gain identification algorithms in the control law.

It is understandably important to clearly separate the study of stability from the study of the performance of the control. While the convergence of an adaptive control to an optimal control will always depend on a certain number of statistical properties of the disturbance acting on the system, the property of stability of the closed-loop system should be established for a large set of disturbances in order to be able to conclude a good robustness of the control. This paper is only concerned with the stability analysis of adaptively controlled systems in the case of bounded disturbances. Since no attempt is made to use the possible statistical properties of the disturbance, we do not try to achieve the convergence of the adaptive control to an optimal control. We only require that the adaptive control behave well when the amplitude of the disturbance is small compared to the amplitude of the useful signal.

To our knowledge, B. Egardt is the first to perform an analysis of stability in the case of bounded disturbances [11,12]. A shorter proof of some of the results obtained by B. Egardt is proposed here using a different approach. One of the main interests of this approach is that it allows an analysis of stability of not-necessarily minimum phase systems. This approach in its main features resumes the approach that we have used in the disturbance-free case [5,13]. The basic idea is that the stability of the adaptively controlled system can be quickly established as soon as the identification algorithm associated with the adaptive control verifies certain properties (easily verified in the disturbance-free case). Furthermore, this approach has the advantage stressed also in Ref. 14, of being effective for a large set of control methods.

In order to make this article shorter and easier to read, we restrict our analysis of stability in two ways:

(1) We only consider "indirect" adaptive control schemes ("indirect" because we identify the parameters of the process in order to calculate the control contrary to "direct" schemes where the controller parameters are directly identified). Since "direct" and "indirect" schemes are often closely related for minimum phase systems [11], a result of stability obtained for an indirect scheme can usually be extended to corresponding direct schemes without great difficulty.

Furthermore, to our knowledge, no general direct adaptive control scheme has yet been proposed for non-minimum phase systems.

(2) Usually the goal of the control is to make the process output coincide, according to a certain criterion, with a reference signal (that can be generated by a linear reference model). We shall only consider the regulator problem, i.e., when the signal of reference is equal to zero. It is well known that the results of the stability analysis remain unchanged for any uniformly bounded signal of reference.

This paper is organized as follows. In Section 1 we state the problem and give several equivalent representations of the system which we want to control. In Section 2, we describe the three properties which determine the choice of the identification algorithm to be used in the control law. These three properties allow in many cases to perform rapidly an analysis of the stability of the adaptively controlled system. An example of an identification algorithm verifying these properties is given. A result of stability for adaptively controlled minimum phase systems is described in Section 3. The analysis of stability is extended in Section 4 to non-minimum phase systems and is applied to a recursive adaptive control algorithm: COMAD ([5,13,15,16]) that we show ensures the stability of the closed-loop system under a condition little restrictive in practice. All proofs of the stated results are found in the appendices A, B, C and D.

1. STATEMENT OF THE PROBLEM

We assume that the system to be controlled is a discrete SISO time-invariant linear system and that it can be represented in the form:

$$y_t = \frac{q^{-1}\bar{B}(q^{-1})u_t + \varepsilon_t}{1-q^{-1}\bar{A}(q^{-1})} \tag{1.1}$$

with:

$$\bar{A}(q^{-1}) = a_1 + a_2q^{-1} + \dots + a_nq^{-n+1}$$

$$\bar{B}(q^{-1}) = b_1 + b_2q^{-1} + \dots + b_nq^{-n+1}$$

$$q^{-1}y_t = y_{t-1}$$

u_t: system input at time t

y_t: system output at time t

ε_t: disturbance acting on the system such that: $|\varepsilon_t| < M, \ \forall t$

The only assumption made on the system is:

$\underline{A_1}$: The system is stabilizable, i.e., polynomials $\bar{B}(z)$ and $(1-z\bar{A}(z))$ have no common root inside the unit disk.

In the sequel we also assume that:

$\underline{A_2}$: We know an upper bound, n, of the degrees of polynomials $\bar{A}$ and $\bar{B}$.

Notice that the disturbance sequence $\{\varepsilon_t\}$ can be any uniformly bounded sequence of real numbers. No assumption is therefore made concerning the statistical properties of the disturbance.

Relation (1.1) can also be written:

$$y_t = \phi_{t-1}^T\theta + \varepsilon_t$$

$$\theta^T = [a_1 \dots a_n \ b_1 \dots b_n]$$

$$\phi_{t-1}^T = [y_{t-1} \dots y_{t-n} \ u_{t-1} \dots u_{t-n}] \tag{1.2}$$

or in the observable canonical state form:

$$x_{t+1} = Ax_t + Bu_t + K\varepsilon_t$$

$$y_t = Cx_t + \varepsilon_t \tag{1.3}$$

with: $A = S + KC$

$$B^T = [b_1 \; \dots \; b_n]$$

$$K^T = [a_1 \; \dots \; a_n]$$

$$C = [10 \; \dots \; 0]$$

$$S = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ \vdots & & & \ddots & 1 \\ 0 & \dots & \dots & \dots & 0 \end{bmatrix}$$

We verify easily that relation (1.3) can also be written:

$$x_t = H\,\phi_{t-1}$$

$$y_t = Cx_t + \varepsilon_t \tag{1.4}$$

with:

$$H = \begin{bmatrix} a_1 & \dots & \dots & a_n & b_1 & \dots & \dots & b_n \\ a_2 & \dots & a_n & 0 & b_2 & \dots & b_n & 0 \\ \vdots & \iddots & \iddots & \vdots & \vdots & \iddots & \iddots & \vdots \\ a_n & 0 & \dots & 0 & b_n & 0 & \dots & 0 \end{bmatrix}$$

The four representations (1.1)-(1.4) of the system are equivalent and can therefore be used indifferently.

Since we restrict ourselves to indirect schemes of adaptive control, we identify the parameter vector θ. The identification algorithm is studied in the next section.

2. THE IDENTIFICATION ALGORITHM

In order to identify the system parameter vector θ, we consider a recursive identification algorithm in the classical form:

$$\theta_t = \theta_{t-1} + P_{t-1}\phi_{t-1}(y_t - \phi_{t-1}^T\theta_{t-1}) , \tag{2.1}$$

where: θ_t designates the current estimate of θ,

P_{t-1} is the gain of the identification algorithm.

In order to be able to perform the stability analysis, the gain P_{t-1} is calculated so that the identification algorithm verifies the three following properties:

P_1: $\quad ||\theta_t|| < M_\theta , \quad \forall t$

P_2: $\quad |y_t - \phi_{t-1}^T\theta_{t-1}| < M_1 + \alpha_t ||\phi_{t-1}||$

with $\alpha_t \geq 0, \quad \lim_{t\to+\infty} \alpha_t = 0, \quad M_1 \geq 0$,

P_3: $\quad ||\theta_t - \theta_{t-1}|| \; ||\phi_{t-1}|| < M_2 + \beta_t ||\phi_{t-1}||$

with $\beta_t \geq 0, \quad \lim_{t\to+\infty} \beta_t = 0, \quad M_2 \geq 0$.

($||\cdot||$ designates the Euclidian norm.)

Notice that these properties are, though less restrictive, the same properties as those demanded in Refs. 5, 13 and 14 for the disturbance-free case.

Let us comment briefly on these properties.

Property P_1 means that the identified vector θ_t is uniformly bounded. This property is a key property and seems to be very important for the stability analysis. We shall see that it prevents the system from diverging faster than exponentially. This maximum divergence rate is very useful for the stability analysis.

Property P_2 ensures that the "prediction error" $(y_t-\phi_{t-1}^T\theta_{t-1})$ stays very small compared with the size of the vector ϕ_{t-1} when this vector becomes large. Since the term $\phi_{t-1}^T\theta_{t-1}$ can be interpreted as the output of an adaptive observer of the system (see Section 4), this property means that the adaptive observer transfer function is very similar to that of the system. Controlling the system is then roughly equivalent to controlling the adaptive observer.

Finally, Property P_3 means that when ϕ_{t-1} becomes large the difference between two consecutive estimates of θ becomes very small. This property allows the control of the time-varying adaptive observer of the system.

Now the question is whether properties P_1-P_3 are realistic and if there exists identification algorithm in the form (1.3) that verify those properties. Indeed, we know that these properties are easily verified by most of the usual identification algorithms in the disturbance-free case [4,6,14,15]; on the other hand, it is not obvious whether these properties are still valid when the system is submitted to any bounded disturbance. We now give an example of a non-decreasing gain identification algorithm which we show in Appendix A verifying the properties P_1-P_3.

<u>Example</u>

This algorithm is in the form (2.1) and the gain P_{t-1} is calculated as follows:

$$P_{t-1} = 0 \quad \text{if} \quad |y_t-\phi_{t-1}^T\theta_{t-1}| < 2M_\varepsilon , \tag{2.2}$$

$$P_{t-1} = \frac{\eta_t}{||\phi_{t-1}||^2+\mu_t} \quad \text{if} \quad |y_t-\phi_{t-1}^T\theta_{t-1}| \geq 2M_\varepsilon , \tag{2.3}$$

where: M_ε is a known upper bound of the amplitude of the disturbance

$0 < \lambda < \eta_t \leq 1$, $\forall t$

$0 < \mu_t \leq M_\mu$, $\forall t$

M_μ being an arbitrary positive real number.

The alternative (2.2) corresponds to what B. Egardt [11,12] calls a "conditional updating." A physical interpretation of this alternative follows. A small error (less than twice the normal amplitude of the disturbance) between the system output and the prediction output $\hat{y}_{t|t-1} = \phi^T_{t-1}\theta_{t-1}$ may indicate that the identified vector θ_{t-1} is a good estimate of θ and that it is not therefore necessary to modify this estimate. This leads to choose $P_{t-1} = 0$. In fact, the alternative (2.2) corresponds to a precautionary measure. Indeed, when the error $(y_t - \phi^T_{t-1}\theta_{t-1})$ is small and when the disturbance ε_t does not have good statistical properties, this error is no longer liable to bring out information in order to improve the identification of θ. To insist on using this error to identify θ may then have the opposite effect of deteriorating the identification of θ and consequently spoiling the adaptive control based on this identification.

On the other hand, a conditional updating like (2.2) presents two major drawbacks: (1) we have to know an upper bound M_ε of the amplitude of the disturbance. In certain circumstances, this condition can be unsatisfactory, and (2) a conditional updating procedure prevents an improvement of the identification when the disturbance ε_t has good statistical properties.

Let us recall that the identification algorithm computed from (2.1), (2.2) and (2.3) is only an example of an algorithm that verifies the properties P_1-P_3. It is still an open question whether or not there exist algorithms without conditional updating that verify those properties.

A fairly simple reasoning seems nevertheless to indicate that it is prudent to include a mechanism in the identification algorithm that ensures the boundedness of θ_t (at least when the gain of the algorithm is a non-decreasing gain). Indeed from relation (2.1) we have:

$$||\theta_t||^2 = ||\theta_{t-1}||^2 + 2w_t\ \theta^T_{t-1}P_{t-1}\phi_{t-1} + \phi^T_{t-1}P_{t-1}P_{t-1}\phi_{t-1}\ w^2_t \tag{2.4}$$

with

$$w_t = y_t - \phi^T_{t-1}\theta_{t-1} .$$

Let us then assume that the control u_t is computed in order to have:

$$\theta^T_{t-1}P_{t-1}\phi_{t-1} = 0 , \qquad \forall t . \tag{2.5}$$

Such a control is not unrealistic since, when P_{t-1} is a scalar gain, the equality (2.5) is verified as soon as:

$$\theta^T_{t-1}\phi_{t-1} = 0 , \qquad \forall t . \tag{2.6}$$

Relation (2.6) corresponds to a classical "dead-beat control strategy" that will be used in the next section.

Thus, according to relations (2.4) and (2.5):

$$||\theta_t||^2 = ||\theta_{t-1}||^2 + \phi^T_{t-1}P_{t-1}P_{t-1}\phi_{t-1}\ w^2_t . \tag{2.7}$$

Relation (2.7) shows that in this particular case the norm of θ_t is always increasing and tends towards $+\infty$ if $(\phi^T_{t-1}P_{t-1}P_{t-1}\phi_{t-1}\ w^2_t)$ does not tend to zero.

When the gain P_{t-1} is non-decreasing, there is no reason for the term $(\phi^T_{t-1}P_{t-1}P_{t-1}\phi_{t-1}\ w^2_t)$ to converge to zero or even to become small. Therefore, if no specific measure, like the alternative (2.2), is taken to ensure the boundedness of θ_t, there is a risk of θ_t diverging.

3. STABILITY ANALYSIS OF ADAPTIVELY CONTROLLED SYSTEMS WITH CANCELLATION OF THE ZEROS OF THE TRANSFER FUNCTION

There exists in the literature numerous techniques to calculate an adaptive control for a time-invariant linear system. We shall consider the simplest of them which is based on the minimization of the cost:

$$J_o = \lim_{T\to+\infty} \frac{1}{T} \sum_{t=0}^{T} y_t^2 .$$

In the disturbance-free case this strategy consists of trying to directly set y_t to zero (dead-beat control). In a stochastic design, the goal is to minimize the system output variance (self-tuning regulator [17]). Most of the adaptive control techniques studied in the literature are based upon this strategy or upon a similar strategy (minimization of the filtered output in the case of model reference adaptive control). It is well known that this type of strategy leads to the cancellation of the zeros of the system transfer function. In order to ensure the stability of the control, two additional assumptions, A_3 and A_4, must then be fulfilled.

<u>A_3</u>: The system has a <u>known</u> number, k, of delays (the k first parameters of polynomial $\bar{B}(q^{-1})$ are equal to zero).

The system (1.1) can then be written:

$$y_t = \frac{q^{-(1+k)}\,\bar{B}'(q^{-1})\,u_t + \varepsilon_t}{1-q^{-1}\,\bar{A}(q^{-1})} , \tag{3.1}$$

with: $\bar{B}'(q^{-1}) = b_{k+1} + b_{k+2}q^{-1} + \ldots + b_n q^{-n+k+1}$,

or also:

$$y_t = \theta^T \phi_{t-1} + \varepsilon_t , \tag{3.2}$$

with: $\theta^T = [a_1 \ldots a_n \; b_{k+1} \ldots b_n]$,

$$\phi_{t-1}^T = [y_{t-1} \ldots y_{t-n} \; u_{t-k-1} \ldots u_{t-n}] .$$

A_4: All roots of polynomial $\overline{B}(z)$ which are not equal to zero are outside the unit disk.

We show in the next section that these two assumptions can be avoided if we slightly modify our control strategy.

A Dead-Beat Adaptive Control Design

When no information is easily available about the disturbance ε_t, or when no attempt is made to use the possible statistical properties of the disturbance, a natural prediction of the output y_t at time $t-1$ is:

$$\hat{y}_{t|t-1} = \theta^T_{t-1}\phi_{t-1} \ . \tag{3.3}$$

A natural prediction of y_t at time $t-2$ is:

$$\begin{aligned}\hat{y}_{t|t-2} = {} & a_{1,t-2}\, \hat{y}_{t-1|t-2} + a_{2,t-2}\, y_{t-2} + \dots + a_{n,t-2}\, y_{t-n} \\ & + b_{k+1,t-2}\, u_{t-k-1} + \dots + b_{n,t-2}\, u_{t-n}\end{aligned} \tag{3.4}$$

($a_{i,t}$ and $b_{j,t}$ designate the respective estimates at time t of the parameters a_i and b_j.)

A natural prediction of y_t at time $t-k-1$ is:

$$\begin{aligned}\hat{y}_{t|t-k-1} = {} & a_{1,t-k-1}\, \hat{y}_{t-1|t-k-1} + a_{2,t-k-1}\, \hat{y}_{t-2|t-k-1} + \dots \\ & + a_{k+1,t-k-1}\, y_{t-k-1} + \dots + a_{n,t-k-1}\, y_{t-n} \\ & + b_{k+1,t-k-1}\, u_{t-k-1} + \dots + b_{n,t-k-1}\, u_{t-n} \ .\end{aligned} \tag{3.5}$$

A dead beat control strategy leads to compute the control that realizes:

$$\hat{y}_{t|t-k-1} = 0 \ , \qquad \forall t \ . \tag{3.6}$$

Indeed, if $\theta_t = \theta$ and if $\varepsilon_t = 0$, then $\hat{y}_{t|t-k-1} = y_t = 0$.

The adaptive control expression is therefore obtained by writing that the right-sided term of relation (3.5) is equal to zero, which gives:

$$u_{t-k-1} = -\frac{1}{b_{k+1,t-k-1}}\,[a_{1,t-k-1}\hat{y}_{t-1|t-k-1} + \dots + a_{k+1,t-k-1}y_{t-k-1} + \dots$$
$$+ a_{n,t-k-1}y_{t-n} + b_{k+2,t-k-1}u_{t-k-2} + \dots + b_{n,t-k-1}u_{t-n}] \tag{3.7}$$

Notice that this control is only defined if the identified parameter $b_{k+1,t}$ is not equal to zero.

In order to establish a result of stability, we impose in fact a stronger condition on this parameter:

C_1: There exists a positive real ε so that: $|b_{k+1,t}| > \varepsilon$, $\forall t$.

When the sign of b_{k+1} and a lower bound of $|b_{k+1}|$ are known, we show in the appendix A that a very simple mecanism can be added to the identification algorithm (2.1)-(2.3) in order to ensure the realization of condition C_1.

By identifying in a special way the first — non-equal to zero — control parameter of the system, condition C_1 can be replaced by another condition bearing upon the size of this parameter [11]. Condition C_1 is more widely commented upon in Ref. 3.

The main result of this section is stated in the following lemma.

Lemma 1:

Under the assumptions A_1-A_4 and under the condition C_1:

Given: • The system (1.1)

• An identification algorithm which verifies the properties P_1-P_3.

Then: (i) The adaptive control (3.7) ensures the stability of the closed-loop system (all signals are uniformly bounded)

(ii) According to property P_2 of the identifier, there exists a time t_1 so that:

$$\forall t > t_1, \quad |y_t - \theta_{t-1}^T \phi_{t-1}| < M_1 + \varepsilon,$$

where ε is an arbitrary small positive number.

This inequality can be used to obtain an upper bound of the output y_t.

For example if $k = 0$, $\hat{y}_{t|t-1} = \theta^T_{t-1}\phi_{t-1} = 0$ and therefore:

$$|y_t| < M_1 + \varepsilon \quad (t > t_1) .$$

The result (ii) of the lemma will be a little more precise if we use the specific identification algorithm defined by relations (2.1)-(2.3). We then show that there exists a time t_1 so that $(t > t_1)$:

- $\theta_t = \theta_{t_1} = \text{constant}$
- $|y_t - \theta^T_{t-1}\phi_{t-1}| < 2M_\varepsilon$
- $k = 0 \Rightarrow |y_t| < 2M_\varepsilon$

  $k = 1 \Rightarrow |y_t| < 2M_\varepsilon(1+ a_{1,t_o})$.

  etc.

For simplicity, we give the proof of lemma 1 in Appendix B for $k=1$; its extension to general k is straightforward but lengthy.

Remarks

- The principle of the proof of lemma 1 can be extended to any traditional adaptive control scheme based on the cancellation of the zeros of the system transfer function.

- We can verify that the result (i) of lemma 1 is very similar to the theorem 4.3 of B. Egardt in Ref. 11. But thanks to properties P_1-P_3, that we show are verified by the identifier, our proof is more direct and shorter. On the other hand, we have not been able to establish these properties when the conditional updating is replaced by a projection mechanism that maintains automatically the estimate θ_t in a compact set. In this way, our proof is less general than that proposed by B. Egardt. The main interest of our approach is that it allows a straightforward extension of the stability analysis to adaptively controlled non-minimum phase systems. This is done in the next section.

4. STABILITY ANALYSIS OF ADAPTIVELY CONTROLLED NON-MINIMUM PHASE SYSTEMS

In this section we do not make the assumption A_3 and A_4. The system can therefore be non-minimum phase and have an arbitrary unknown number of time delays.

The approach that we propose for studying the adaptive control of non-minimum phase systems is expanded upon in Refs. 5, 13, 15 and 16 in the disturbance-free case. It consists in first introducing an adaptive observer of the system to be controlled and then controlling the adaptive observer in order to control the system. The explicit formulation of this observer is not necessary in the disturbance-free case [14]. On the other hand, it is well known that it is important to determine an observer of the system when the system is subjected to stochastic disturbances. Let us recall as an example a classical result of the quadratic cost optimal control theory for linear systems: the optimal control is a linear combination of the components of the best least-squares estimate of the state of the system. An application of this statement to determine the adaptive control law can be found in Ref. 15.

The Adaptive Observer

The role of this observer is to bring out an estimation of the state of the system. When no attempt is made to use the statistical properties of the disturbance or when the disturbance does not have good statistical properties, it is natural to consider the state x_t defined in relation (1.3). A corresponding adaptive observer is obtained by replacing the uknown parameters by their estimates, which leads to:

$$\hat{x}_{t+1} = A_t\hat{x}_t + B_tu_t + K_t(y_t - C\hat{x}_t) ,$$

$$\hat{y}_t = C\hat{x}_t , \qquad (4.1)$$

with: $\quad A_t = A(\theta_t)$, $\quad B_t = B(\theta_t)$, $\quad K_t = K(\theta_t)$.

We easily verify that the observer (4.1) can also be written:

$$\hat{x}_t = H'_t \phi_{t-1} \quad ; \quad y_t = C\hat{x}_t$$

$$H'_t = \begin{bmatrix} a_{1,t-1} & a_{2,t-2} & \cdots & a_{n,t-n} & b_{1,t-1} & b_{2,t-2} & \cdots & b_{n,t-n} \\ a_{2,t-1} & \cdots & a_{n,t-n+1} & 0 & b_{2,t-1} & \cdots & b_{n,t-n+1} & 0 \\ \vdots & \iddots & \iddots & \vdots & \vdots & \iddots & \iddots & \vdots \\ a_{n,t-1} & 0 & \cdots & 0 & b_{n,t-1} & 0 & \cdots & 0 \end{bmatrix} \qquad (4.2)$$

($a_{i,t}$ and $b_{j,t}$ designate the respective estimates of a_i and b_j at time t.)

A variant of this observer consists in using in the computation of $\hat{x}_t$ only the last identified parameter vector θ_t, which leads to:

$$\hat{x}_t = H_t \phi_{t-1} ,$$

$$\hat{y}_t = C\hat{x}_t , \qquad (4.3)$$

with $\quad H_t = H(\theta_t)$.

(The matrix H is defined in relation (1.4).)

The choice of the observer (4.3) instead of (4.2) may improve the estimation of x_t as well as the performances of the adaptive control computed with respect to $\hat{x}_t$. Nevertheless, as a consequence of property P_3 of the identifier, this choice does not affect the stability analysis.

Having determined the adaptive observer of the system, we can state the following lemma whose proof is given in Appendix C.

Lemma 2:

Under the assumptions A_1-A_2:

Given: • The system (1.1)

• An identification algorithm verifying the properties P_1-P_3

• The adaptive observer (4.1) (or (4.3)).

If we can determine a sequence $\{L_t\}$ of uniformly bounded matrices so that the system $z_{t+1} = (A_t - B_t L_t) z_t$ is uniformly exponentially stable, then:

(i) The control:

$$u_t = -L_t \hat{x}_t \tag{4.4}$$

ensures the stability of the system (all signals are uniformly bounded).

(ii) According to property P_2 of the identifier, there exists a time t_1 so that:

$$\forall t > t_1 , \qquad |y_t - \theta^T_{t-1} \phi_{t-1}| < M_1 + \varepsilon$$

(where ε is an arbitrary small positive number).

The result (ii) will be a little more precise if we use the specific identification algorithm defined by relations (2.1)-(2.3). We then show that there exists a time t_1 so that $(t > t_1)$:

• $\theta_t = \theta_{t_1} = \text{constant}$

• $|y_t - C\hat{x}_t| < 2M_\varepsilon$.

Lemma 2 states that any way of computing the sequence $\{L_t\}$ so that the matrix $(A_t - B_t L_t)$ is uniformly exponentially stable solves the problem of the stability of the system. The important question we then have to answer is the following: Do we know how to calculate a sequence $\{L_t\}$

verifying this property? If the matrices A_t and B_t were constant, we would be brought back to a classical control problem of a time-invariant linear system (the adaptive observer) with known parameters. This problem can be solved in many ways: pole placement, minimization of a quadratic criterion, etc. Unfortunately, θ_t varies in time and setting the eigenvalues of the matrix $(A_t-B_tL_t)$ to fixed values inside the unit disk is generally no longer sufficient to ensure the uniform exponential stability of this matrix. Nevertheless, the property P_3 of the identifier notably improves the statement of the problem. The following reasoning will be formalized in Appendix D for a particular choice of the sequence $\{L_t\}$. According to property P_3, when ϕ_t becomes large, $||\theta_t-\theta_{t-1}||$ becomes very small. Therefore, when ϕ_t is large the matrices A_t and B_t are almost constant. Any classical control method (pole placement, minimization of a quadratic criterion, etc.) then allows us to calculate a matrix L_t so that the matrix $A_t-B_tL_t$ has fixed (or almost fixed) eigenvalues inside the unit disk and is locally (as long as ϕ_t stays large) uniformly exponentiably stable. This property of the matrix $(A_t-B_tL_t)$ in becoming uniformly exponentially stable when ϕ_t becomes large is sufficient to ensure the boundedness of ϕ_t.

We now give an example of determination of the sequence $\{L_t\}$ based on a quadratic cost control strategy.

Example: The Adaptive Controller COMAD

This control has been designed in regard to minimizing a criterion of the type: $J_\lambda = \lim_{T\to+\infty} \frac{1}{T}\sum_{t=0}^{T} y_t^2 + \lambda u_t^2$, $\lambda > 0$.

Notice that the introduction of the term (λu_t^2) $(\lambda>0)$ in this criterion allows us not to make the assumptions A_3 and A_4 since the minimization of this criterion no longer involves the cancellation of the zeros of the system transfer function.

The control COMAD is calculated as follows:

$$u_t = -L_t \hat{x}_t + e_t \tag{4.5}$$

(e_t is an optional term, whose role is explained in Ref. 15, that does not change the stability analysis of the closed-loop system) with:

$$e_t = -\frac{B_t^T R_t K_t}{B_t^T R_t B_t + \lambda} (y_t - C\hat{x}_t) \tag{4.6}$$

$$L_t = \frac{B_t^T R_t A_t}{B_t^T R_t B_t + \lambda} \tag{4.7}$$

$$R_{t+1} = C^T C + A_t^T R_t A_t - \frac{A_t^T R_t B_t B_t^T R_t A_t}{B_t^T R_t B_t + \lambda} \tag{4.8}$$

We show (Appendix D) that the sequence $\{L_t\}$ of relation (4.7) is uniformly bounded and has good properties if the following condition is fulfilled:

$\underline{C_2}$: There exists an integer r, a positive real $q < 1$, and a sequence $\{G_i\}_{i \in N}$ of uniformly bounded matrices so that:

$$||(A_k - B_k G_k)(A_{k+1} - B_{k+1} G_{k+1}) \ldots (A_{k+r-1} - B_{k+r-1} G_{k+r-1})|| < q < 1$$

$$\text{for } k = 0, r, 2r, \ldots .$$

This condition replaces the condition C_1 of the previous section. The condition C_2 deviates from the usual definition of uniform stabilizability of the pair (A_t, B_t) in that the product of the matrices $(A_i - B_i L_i)$ is made with the index i increasing, instead of decreasing, from the left to the right side. This condition is not therefore more restrictive than the condition of uniform stabilizability commented upon in Ref. 13.

When $||\theta_t-\theta_{t-1}||$ becomes very small, it is straightforward to show that C_2 gives once again the usual condition of uniform stabilizability.

Notice that in order to place the eigenvalues of the matrix $(A_t-B_tL_t)$, a condition like C_2, or even stronger, will always be required [14].

The main result of this paragraph is the following (this result should be compared with lemma 1 of Section 3):

Lemma 3:

Under the assumptions A_1-A_2 and under the condition C_2:

Given:
- The system (1.1)
- An identification algorithm that verifies the properties P_1-P_3
- An adaptive observer ((4.1) or (4.3))

Then: (i) The adaptive control (4.5) ensures the stability of the system (all signals are uniformly bounded).

(ii) According to property P_2 of the identifier, there exists a time t_1 so that:

$$\forall t > t_1 , \quad |y_t-\theta_{t-1}^T\phi_{t-1}| < M_1 + \varepsilon$$

(where ε is an arbitrary small positive number).

The proof of lemma 3 is given in Appendix D.

The result (ii) of this lemma will be a little more precise when using the specific identification algorithm defined by relations (2.1)-(2.3). We show in this case that there exists a time t_1 so that $(t_1 > t_o)$:

- $\theta_t = \theta_{t_1} = \text{constant}$
- $|y_t-C\hat{x}_t| < 2M_\varepsilon$ (M_ε is a known upper bound of the disturbance amplitude)
- The adaptive observer (4.1) (or (4.3)) is:

$$\hat{x}_{t+1} = A_{t_1}\hat{x}_t + B_{t_1}u_t + K_{t_1}(y_t - C\hat{x}_t) \tag{4.9}$$

- and the adaptive control (4.5):

$$u_t = -L_{t_1}\hat{x}_t - \frac{B^T_{t_1} R_{t_1} K_{t_1}}{B^T_{t_1} R_{t_1} B_{t_1} + \lambda}(y_t - C\hat{x}_t) \tag{4.10}$$

Relations (4.9) and (4.10) allow the study of the performances of the adaptive control with respect to the identified parameter vector θ_{t_1}.

CONCLUDING REMARKS

We have proposed a possible approach to perform the stability analysis of an adaptively controlled system when this system is subjected to bounded disturbances. This approach is a natural extension of an approach developed in the disturbance-free case [5,13-16]. It consists of first ensuring that the identification algorithm to be used verifies certain properties (easily verified in the disturbance-free case). The stability analysis of the system then becomes systematic and relatively simple. The main contribution of this approach is that it applies itself not only to traditional adaptive control schemes but also to recent schemes for non-minimum phase systems. The results of uniform boundedness of all signals should be compared to other studies of stability, where either no disturbance is introduced in the problem, or where the disturbances verify very specific statistical properties. It should be possible to improve the approach that we have considered by making the properties P_1-P_3 of the identification algorithm less restrictive. The set of the identification algorithms admissible in the adaptive control law would then be enlarged. It would also be interesting extending the analysis of stability for adaptive control schemes that exploit the statistical properties of the disturbances, without the result of stability depending on the properties of these disturbances.

APPENDIX A

Proposition

Given the system:

$$y_t = \theta^T \phi_{t-1} + \varepsilon_t \tag{A.1}$$

so that: $|\varepsilon_t| < M_\varepsilon$ (M_ε known), the identification algorithm defined by the following relations:

$$\theta_t = \theta_{t-1} + g_{t-1}\phi_{t-1}(y_t - \phi_{t-1}^T \theta_{t-1}) \tag{A.2}$$

$$g_{t-1} = 0 \quad \text{if} \quad |y_t - \phi_{t-1}^T \theta_{t-1}| < 2M_\varepsilon \tag{A.3}$$

$$g_{t-1} = \frac{\eta_t}{||\phi_{t-1}||^2 + \mu_t} \quad \text{if} \quad |y_t - \phi_{t-1}^T \theta_{t-1}| \geq 2M_\varepsilon \tag{A.4}$$

$$(0 < \lambda < \eta_t \leq 1 \; ; \quad 0 < \mu_t < M_\mu)$$

verifies the three properties P_1-P_3 of Section 2.

Proof

Verification of P_1:

Define: $\tilde{\theta}_t = \theta - \theta_t$, according to (A.2):

$$\tilde{\theta}_t = (I - g_{t-1}\phi_{t-1}\phi_{t-1}^T)\tilde{\theta}_{t-1} - g_{t-1}\phi_{t-1}\varepsilon_t \tag{A.5}$$

Relation (A.5) allows to establish rapidly:

$$||\tilde{\theta}_t||^2 = ||\tilde{\theta}_{t-1}||^2 - g_{t-1}(1 - g_{t-1}||\phi_{t-1}||^2)(\phi_{t-1}^T\tilde{\theta}_{t-1} + \varepsilon_t)^2$$
$$- g_{t-1}(\phi_{t-1}^T\tilde{\theta}_{t-1})^2 + g_{t-1}\varepsilon_t^2 \tag{A.6}$$

By using the definition of the gain g_{t-1}, it is straightforward to show:

$$1 - g_{t-1}||\phi_{t-1}||^2 \geq 0 \tag{A.7}$$

and:

$$g_{t-1}[(\phi^T_{t-1}\tilde{\theta}_{t-1})^2 - \varepsilon^2_t] \geq 0 \tag{A.8}$$

From relations (A.6), (A.7) and (A.8) it results:

$$||\tilde{\theta}_t||^2 \leq ||\tilde{\theta}_{t-1}||^2 \tag{A.9}$$

The positive sequence $\{||\tilde{\theta}_t||\}$ being decreasing, this sequence converges and:

$$||\tilde{\theta}_t|| \leq ||\tilde{\theta}_o|| , \qquad \forall t \tag{A.10}$$

Property P_1 follows.

<u>Verification of P_2:</u>

If the inequality: $|y_t - \theta^T_{t-1}\phi_{t-1}| < 2M_\varepsilon$ is always true, property P_2 is trivially verified.

If $|y_t - \theta^T_{t-1}\phi_{t-1}| \geq 2M_\varepsilon$ occurs a finite number of times, P_2 is still trivially verified.

As a last possible alternative, let us therefore assume that $|y_t - \theta^T_{t-1}\phi_{t-1}| \geq 2M_\varepsilon$ occurs infinitely often, i.e., the gain g_{t-1} is infinitely often different from zero. For simplicity, we can assume without loss of generality that the inequality $|y_t - \theta^T_{t-1}\phi_{t-1}| \geq 2M_\varepsilon$ is always true.

The sequence $\{||\tilde{\theta}_t||^2\}$ being decreasing, we conclude from relation (A.6) that:

$$\sum_{t=1}^{+\infty} g_{t-1}[(\tilde{\theta}^T_{t-1}\phi_{t-1})^2 - \varepsilon^2_t] < ||\tilde{\theta}_o||^2 \quad (\text{with } g_{t-1} > 0) \tag{A.11}$$

Therefore:

$$\lim g_{t-1}[(\tilde{\theta}^T_{t-1}\phi_{t-1})^2 - \varepsilon_t^2] = 0 \tag{A.12}$$

Define the positive sequence $\{\alpha_t\}$ as follows:

$$g_{t-1}[(\tilde{\theta}^T_{t-1}\phi_{t-1})^2 - \varepsilon_t^2] = \frac{\alpha_t^2\, \eta_t}{2} \tag{A.13}$$

Since: $0 < \lambda < \eta_t$, (A.12) involves: $\lim\limits_{t\to+\infty} \alpha_t = 0$.

Relation (A.13) can also be written by using the expression of the gain g_{t-1}:

$$(\tilde{\theta}^T_{t-1}\phi_{t-1})^2 - \varepsilon_t^2 = \frac{\alpha_t^2}{2}\,[||\phi_{t-1}||^2 + \mu_t] \tag{A.14}$$

or also:

$$2[(\tilde{\theta}^T_{t-1}\phi_{t-1})^2 + \varepsilon_t^2] = 4\varepsilon_t^2 + \alpha_t^2[||\phi_{t-1}||^2 + \mu_t] \tag{A.15}$$

Now:

$$(\tilde{\theta}^T_{t-1}\phi_{t-1} + \varepsilon_t)^2 \le 2[(\tilde{\theta}^T_{t-1}\phi_{t-1})^2 + \varepsilon_t^2] \tag{A.16}$$

(Schwartz inequality) and $\tilde{\theta}^T_{t-1}\phi_{t-1} + \varepsilon_t = y_t - \phi^T_{t-1}\theta_{t-1}$, by definition of y_t.

From (A.15) and (A.16) we have:

$$(y_t - \phi^T_{t-1}\theta_{t-1})^2 \le 4\varepsilon_t^2 + \alpha_t^2[||\phi_{t-1}||^2 + \mu_t] \tag{A.17}$$

Since ε_t and μ_t are bounded, and since $\lim\limits_{t\to+\infty} \alpha_t = 0$, there exists a positive real M_1 so that

$$4\varepsilon_t^2 + \alpha_t^2\,\mu_t < M_1^2$$

From (A.17):

$$(y_t - \phi^T_{t-1}\theta_{t-1})^2 \le M_1^2 + \alpha_t^2||\phi_{t-1}||^2 \le (M_1 + \alpha_t||\phi_{t-1}||)^2 \tag{A.18}$$

which establishes property P_2.

Verification of P_3:

Property P_3 is trivially verified if $|y_t - \theta^T_{t-1}\phi_{t-1}| < 2M_\varepsilon$ since then: $||\theta_t - \theta_{t-1}|| = 0.$

If $|y_t - \theta^T_{t-1}\phi_{t-1}| \geq 2M_\varepsilon$, relation (A.2) involves:

$$||\theta_t - \theta_{t-1}|| \; ||\phi_{t-1}|| = \frac{\eta_t ||\phi_{t-1}||^2}{||\phi_{t-1}||^2 + \mu_t} |y_t - \phi^T_{t-1}\theta_{t-1}|| \quad (\eta_t \leq 1) \tag{A.19}$$

and therefore:

$$||\theta_t - \theta_{t-1}|| \; ||\phi_{t-1}|| \leq |y_t - \phi^T_{t-1}\theta_{t-1}| \tag{A.20}$$

From property P_2 and by setting: $M_2 = M_1$ and $\beta_t = \alpha_t$

$$||\theta_t - \theta_{t-1}|| \; ||\phi_{t-1}|| \leq M_2 + \beta_t ||\phi_{t-1}|| \tag{A.21}$$

$$\text{with} \quad \lim_{t \to +\infty} \beta_t = 0$$

which establishes P_3.

In addition, let us show also the following proposition that is used to precise the result (ii) of lemmas 1-3.

Proposition:

Given the identification algorithm defined by relations (A.2)-(A.4); if $\{\phi_t\}$ is uniformly bounded then the inequality: $|y_t - \phi^T_{t-1}\theta_{t-1}| \geq 2M_\varepsilon$ can only occur a finite number of times. Therefore, there exists a time t_1 so that: $\forall t > t_1$: $|y_t - \phi^T_{t-1}\theta_{t-1}| < 2M_\varepsilon$ and $\theta_t = \theta_{t_1}$.

Proof:

Let us assume that $|y_t - \phi^T_{t-1}\theta_{t-1}| \geq 2M_\varepsilon$ is true infinitely often. To simplify we can assume that this inequality is always verified. Since the sequence $\{||\tilde{\theta}_t||^2\}$ is decreasing, relation (A.6) involves:

$$\sum_{t=1}^{+\infty} g_{t-1}(1-g_{t-1}||\phi_{t-1}||^2)(\phi_{t-1}^T\tilde{\theta}_{t-1}+\varepsilon_t)^2 < ||\tilde{\theta}_o||^2 \qquad (A.22)$$

Therefore:

$$\lim_{t\to+\infty} g_{t-1}(1-g_{t-1}||\phi_{t-1}||^2)(\phi_{t-1}^T\tilde{\theta}_{t-1}+\varepsilon_t)^2 = 0 \qquad (A.23)$$

If the sequence $\{\phi_{t-1}\}$ is bounded, the terms g_{t-1} and $(1-g_{t-1}||\phi_{t-1}||^2)$ are bounded from below by a strictly positive real number.

Thus, from (A.23):

$$\lim_{t\to+\infty} (\phi_{t-1}^T\tilde{\theta}_{t-1}+\varepsilon_t)^2 = 0$$

that we can also write:

$$\lim_{t\to+\infty} (y_t-\phi_{t-1}^T\theta_{t-1})^2 = 0$$

This contradicts our starting assumption and therefore establishes the proposition.

<u>Mecanism ensuring the realization of condition C_1</u>

We assume that the sign of b_{k+1} is known (positive for example) and that we know a lower bound b_{min} of this parameter.

To ensure the realization of condition C_1 we can, for example, add to the identification algorithm (A.2)-(A.4) the following mecanism:

At time t:. θ_t is calculated according to (A.2)-(A-4)

$$\begin{aligned} &. \text{ if } b_{k+1,t} \geq \frac{b_{min}}{2}, \; b_{k+1,t} \text{ is unchanged.} \\ &. \text{ if } b_{k+1,t} < \frac{b_{min}}{2}, \; b_{k+1,t} \text{ is replaced by } b_{min}. \end{aligned} \qquad (A.24)$$

We have already shown that without any additional mecanism the positive sequence $\|\tilde{\theta}_t\|^2$ is decreasing. Since:

$$\|\tilde{\theta}_t\|^2=(a_1-a_{1,t})^2+(a_2-a_{2,t})^2+\ldots+(a_n-a_{n,t})^2+(b_{k+1}-b_{k+1,t})^2+$$

$$+\dots+(b_n-b_{n,t})^2 \qquad \text{(A.25)}$$

each time that $b_{k+1,t}$ is replaced by b_{min} (because $b_{k+1,t} < \frac{b_{min}}{2}$) then:

$$\|\tilde{\theta}_t\|^2 = \text{previous } \|\tilde{\theta}_t\|^2 + (b_{k+1}-b_{min})^2 - (b_{k+1}-b_{k+1,t})^2$$

$$\leq \|\tilde{\theta}_{t-1}\|^2 + (b_{k+1}-b_{min})^2 - (b_{k+1}-b_{k+1,t})^2 \qquad \text{(A.26)}$$

Furthermore:

$$(b_{k+1}-b_{k+1,t}) = \underset{\geq 0}{(b_{k+1}-b_{min})} + \underset{> \frac{b_{min}}{2}}{(b_{min}-b_{k+1,t})}$$

and therefore:

$$(b_{k+1}-b_{k+1,t})^2 > (b_{k+1}-b_{min})^2 + \frac{b_{min}^2}{4} \qquad \text{(A.27)}$$

From (A.26) and (A.27):

$$\|\tilde{\theta}_t\|^2 \leq \|\tilde{\theta}_{t-1}\|^2 - \frac{b_{min}^2}{4} \qquad \text{(A.28)}$$

(A.28) is true each time that θ_t is modified according to the mecanism (A.24). Otherwise we only have:

$$\|\tilde{\theta}_t\|^2 \leq \|\tilde{\theta}_{t-1}\|^2$$

Since $\|\tilde{\theta}_t\|^2$ is positive, (A.28) can only be true a finite number of times which means that, after a finite time t_0, $b_{k+1,t}$ stays larger than $\frac{b_{min}}{2}$.

APPENDIX B

We give the proof of lemma 1 for $k = 1$.

The equation of the system is:

$$y_t = \theta^T \phi_{t-1} + \varepsilon_t$$

$$\theta^T = [a_1 \ \dots \ a_n \ b_2 \ \dots \ b_n]$$

$$\phi_{t-1} = [y_{t-1} \ \dots \ y_{t-n} \ u_{t-2} \ \dots \ u_{t-n}] \tag{B.1}$$

The control that we chose realizes: $\hat{y}_{t|t-2} = 0$. We verify easily:

$$u_{t-2} = - \frac{1}{b_{2,t-2}} [a_{1,t-2} \theta^T_{t-2} + \delta^T_t] \phi_{t-2} \tag{B.2}$$

with:

$$\delta^T_t = [a_{2,t-2}; \ \dots; \ a_{n,t-2}; \ 0; \ b_{3,t-2}; \ \dots; \ b_{n,t-2}; \ 0]$$

where $a_{i,t}$ and $b_{j,t}$ designate the respective estimates of a_i and b_j at time t.

According to relation (B.2), condition C_1 and Property P_1 of the identifier, there exists a positive real M_u so that:

$$|u_{t-2}| < M_u \, ||\phi_{t-2}|| \tag{B.3}$$

According to (B.1) we have also:

$$|y_{t-1}| < ||\theta|| \ ||\phi_{t-2}|| + M_\varepsilon \tag{B.4}$$

According to relations (B.1), (B.3) and (B.4) there exists a positive real M_ϕ so that:

$$||\phi_{t-1}|| < M_\phi(||\phi_{t-2}|| + 1) \tag{B.5}$$

This relation means that the sequence $||\phi_t||$ cannot diverge faster than exponentially.

Now, by forming the difference $(y_t - \hat{y}_{t|t-2})$ we obtain:

$$y_t - \hat{y}_{t|t-2} = (y_t - \theta^T_{t-1}\phi_{t-1}) + (\theta_{t-1} - \theta_{t-2})^T\phi_{t-1} - a_{1,t-2}(y_{t-1} - \theta^T_{t-2}\phi_{t-2}) \tag{B.6}$$

Since $\hat{y}_{t|t-2} = 0$, by using the properties P_1-P_3 of the identifier and relation (B.5), we obtain the following inequality:

$$|y_t| < M_y + \alpha_{y,t}||\phi_{t-1}|| + \beta_{y,t}||\phi_{t-2}|| \quad (\forall t) \tag{B.7}$$

with: $\quad M_y > 0 \,, \quad \lim\limits_{t \to +\infty} \alpha_{y,t} = 0 \,, \quad \lim\limits_{t \to +\infty} \beta_{y,t} = 0$

According to (B.5) and (B.7) we have also:

$$|y_t| < M'_y + \alpha'_{y,t}||\phi_{t-2}|| \tag{B.8}$$

with: $\quad M'_y > 0$ and $\lim\limits_{t \to +\infty} \alpha'_{y,t} = 0.$

Now according to (B.1):

$$u_{t-1} = \frac{1}{b_2} y_{t+1} - \frac{a_1}{b_2} y_t - \frac{1}{b_2} [a_2 \ldots a_n \; 0 \; b_3 \ldots b_n \; 0]\phi_{t-1} - \frac{\varepsilon_{t+1}}{b_2} \tag{B.9}$$

By using the definition of ϕ_t and relation (B.9) we can write:

$$\phi_t = R\,\phi_{t-1} + S_t \tag{B.10}$$

with:

$$R = \left[\begin{array}{c|c|c|c}
\multicolumn{4}{c}{0} \\
\hline
I_{n-1} & 0 & \multicolumn{2}{|c}{0} \\
\hline
\multicolumn{2}{c|}{L_1} & \multicolumn{2}{c}{L_2} \\
\hline
\multicolumn{2}{c|}{0} & I_{n-2} & 0
\end{array}\right] \qquad (I_n: \text{ matrix identity of order } n)$$

$$L_1 = -\frac{1}{b_2} [a_2 \ldots a_n \; 0]$$

$$L_2 = -\frac{1}{b_2}[b_3 \ldots b_n \ 0]$$

$$S_t = \begin{bmatrix} y_t \\ 0 \\ \frac{1}{b_2} y_{t+1} - \frac{a_1}{b_2} y_t - \frac{\varepsilon_{t+1}}{b_2} \\ 0 \end{bmatrix}$$

The set of the eigenvalues of R is composed of zeros and of the set of the roots of polynomial $\bar{B}'(z) = b_2 + b_3 z + \ldots + b_n z^{n-2}$. Since we have assumed that this polynomial was minimum phase (assumption A_4), the matrix R is asymptotically exponentially stable.

Define now the augmented vector:

$$\Phi_t = \begin{bmatrix} \phi_t \\ \phi_{t-1} \end{bmatrix} \qquad \text{(B.11)}$$

According to (B.8) and (B.10) we have:

$$\Phi_t = \Gamma\, \Phi_{t-1} + \mathscr{S}_t \ ; \quad \Gamma = \left[\begin{array}{c|c} R & 0 \\ \hline I & 0 \end{array}\right] \ : \ \text{matrix exponentially stable since } R \text{ is exponentially stable} \qquad \text{(B.12)}$$

$$||\mathscr{S}_t|| < M_\Phi + \mu_t ||\Phi_{t-1}|| \qquad \left(\mathscr{S}_t = \begin{bmatrix} S_t \\ 0 \end{bmatrix}\right) \qquad \text{(B.13)}$$

with: $M_\Phi > 0$ and $\lim_{t \to +\infty} \mu_t = 0$.

It results from relations (B.12) and (B.13) that the sequence $\{\Phi_t\}$ is uniformly bounded as therefore all signals.

APPENDIX C

We give the proof of lemma 2.

Let us consider the adaptive observer of relation (4.1) (we could also use the observer of relation (4.3)):

$$\hat{x}_{t+1} = A_t\hat{x}_t + B_tu_t + K_t(y_t-C\hat{x}_t) \tag{C.1}$$

We apply the following control to the system:

$$u_t = -L_t\hat{x}_t \tag{C.2}$$

where $\{L_t\}$ is a uniformly bounded sequence so that the matrix $(A_t-B_tL_t)$ is uniformly exponentially stable.

According to (C.1) and (C.2):

$$\hat{x}_{t+1} = (A_t - B_tL_t)\hat{x}_t + K_t(y_t-C\hat{x}_t) \tag{C.3}$$

According to relation (4.2) we also have:

$$\hat{x}_t = H_t' \phi_{t-1} \tag{C.4}$$

where H_t' is a uniformly bounded matrix.

Therefore, according to (C.2) and (C.4):

$$u_t = -L_t H_t' \phi_{t-1} \tag{C.5}$$

and there exists a positive real M_u so that:

$$|u_t| < M_u||\phi_{t-1}|| \quad (\forall t) \tag{C.6}$$

According to relation (1.2) we also have:

$$|y_t| \leq ||\theta|| \; ||\phi_{t-1}|| + M_\varepsilon$$

It results from relations (C.6) and (C.7) that there exist two positive reals M_ϕ and M' so that:

$$||\phi_t|| \leq M_\phi \; ||\phi_{t-1}|| + M' \tag{C.8}$$

For simplicity, we can assume in the sequel that there exists a small positive real so that:

$$||\phi_t|| > \varepsilon \, , \qquad \forall t$$

Then, according to (C.8):

$$||\phi_t|| \leq M'_\phi \; ||\phi_{t-1}|| \quad \text{with} \quad M'_\phi = M_\phi + \frac{M'}{\varepsilon} \quad . \tag{C.9}$$

This relation means that the sequence $\{\phi_t\}$ cannot diverge faster than exponentially.

By taking the first line of the equality (C.4) we have:

$$C\hat{x}_t = \theta^T_{t-1}\phi_{t-1} + \Delta\theta^T_t\phi_{t-1} \tag{C.10}$$

with: $$\Delta\theta^T_t = [0; (a_{2,t-2}-a_{2,t-1}); \ldots; (a_{n,t-n}-a_{n,t-1}); 0;$$
$$(b_{2,t-2}-b_{2,t-1}) \; \ldots \; (b_{n,t-n}-b_{n,t-1})]$$

It is obvious that:

$$||\Delta\theta_t|| \leq \sum_{i=0}^{n-2} ||\theta_{t-i-1}-\theta_{t-i-2}|| \tag{C.11}$$

and therefore:

$$||\Delta\theta^T_t\phi_{t-1}|| \leq \sum_{i=0}^{n-2} ||\theta_{t-i-2}-\theta_{t-i-2}|| \; ||\phi_{t-1}|| \tag{C.12}$$

By using relations (C.9) and (C.12) and the property P_3 of the identifier, we show easily:

$$||\Delta\theta^T_t\phi_{t-1}|| \leq \mu + \alpha_{\mu,t}||\phi_{t-1}|| \tag{C.13}$$

with: $$\mu = \sum_{i=0}^{n-2} M_1 \; M'^{(i+1)}_\phi$$

$$\alpha_{\mu,t} = \sum_{i=0}^{n-2} \alpha_{t-i-1} , \qquad \lim_{t\to+\infty} \alpha_{\mu,t} = 0$$

According to (C.10):

$$y_t - C\hat{x}_t = (y_t - \theta_{t-1}^T \phi_{t-1}) + \Delta\theta_t^T \phi_{t-1} \tag{C.14}$$

and therefore according to relation (C.13) and property P_2 of the identifier:

$$|y_t - C\hat{x}_t| \leq M_e + \alpha_{e,t} \, ||\phi_{t-1}|| \tag{C.15}$$

with: $M_e > 0$ and $\lim_{t\to+\infty} \alpha_{e,t} = 0$

Define now the augmented vector:

$$\Phi_t = \begin{bmatrix} \hat{x}_{t+1} \\ \phi_t \end{bmatrix} \tag{C.16}$$

According to relations (C.2) and (C.3):

$$\Phi_{t+1} = \mathscr{F}_t \, \Phi_t + \mathscr{E}_t \tag{C.17}$$

with:

$$\mathscr{F}_t = \left[\begin{array}{c|c} A_t - B_t L_t & 0 \\ \hline C & \\ \cline{1-1} 0 & \\ \cline{1-1} -L_t & R \\ \cline{1-1} 0 & \end{array}\right]$$

$$R = \left[\begin{array}{c|c|c|c} \multicolumn{4}{c}{0} \\ \hline I_{n-1} & 0 & \multicolumn{2}{c}{0} \\ \hline \multicolumn{4}{c}{0} \\ \hline \multicolumn{2}{c|}{0} & I_{n-1} & 0 \end{array}\right]$$

$$\mathscr{E}_t = \begin{bmatrix} K_t \\ \hline 1 \\ \hline 0 \end{bmatrix} (y_t - C\hat{x}_t)$$

It is shown in Ref. 5 that the uniform exponential stability of the matrix $(A_t - B_t L_t)$ involves the uniform exponential stability of the matrix $\mathscr{F}_t$ (when L_t is bounded).

Moreover, according to relation (C.15):

$$||\mathscr{E}_t|| \leq M_{\mathscr{E}} + \alpha_{\mathscr{E},t} \; ||\Phi_{t-1}|| \tag{C.18}$$

with: $M_{\mathscr{E}} > 0$ and $\lim\limits_{t \to +\infty} \alpha_{\mathscr{E},t} = 0$

It is shown in Ref. 5 that relations (C.17) and (C.18) involve the uniform boundedness of the sequence $\{\phi_t\}$ as therefore the uniform boundedness of all signals.

APPENDIX D

We give the proof of lemma 3.

Since the sequence $\{\theta_t\}$ verifies condition C_2, then according to lemma 1 of Ref. 18 the matricial sequence $\{R_t\}$ (defined in relation (4.8)) is uniformly bounded, as therefore the sequence $\{L_t\}$.

Let us recall that the pair (A_t,B_t) is uniformly stabilizable if there exists an integer s, a real $q < 1$, and a sequence $\{G_i\}$ uniformly bounded, so that:

$$\left\| \prod_{i=k}^{k+s-1} (A_i-B_iG_i) \right\| < q < 1 \quad \text{for} \quad k=0,s,2s,\ldots \tag{D.1}$$

$$\left(\prod_{i=k}^{k+s-1} (A_i-B_iG_i) = (A_{k+s-1}-B_{k+s-1}G_{k+s-1})(A_{k+s-2}-B_{k+2-2}G_{k+s-2}) \cdots \right.$$

$$\left. \cdots (A_k-B_kG_k) \right)$$

We have shown in Appendix C of Ref. 5 that if (A_t,B_t) is uniformly stabilizable and if $\lim_{t\to+\infty} \|\theta_t-\theta_{t-1}\| = 0$, then the sequence $\{L_t\}$, calculated to relations (4.7) and (4.8), verifies:

$$\exists\, s' \in \mathbb{N},\ \exists\, q < 1;\ \left\| \prod_{i=k}^{k+s'-1} (A_i-B_iL_i) \right\| < q < 1,\ k=0,s',2s',\ldots \tag{D.2}$$

In fact, we verify easily that the proof of (D.2) is still valid if $\|\theta_t-\theta_{t-1}\|$ stays very small and is less than a certain positive real ε_1 (that mainly depends on the upper bounds of $\|A_t\|$, $\|B_t\|$ and $\|L_t\|$).

On the other hand, it is obvious that there exists a positive real ε_2 so that if $\|\theta_t-\theta_{t-1}\| < \varepsilon_2$ $(\forall t)$ then the condition C_2 is equivalent to the property of uniform stabilizability of (A_t,B_t). Indeed, if ε_2

is sufficiently small, the matrices A_i and B_i are almost constant on the interval [k,k+s-1] and:

$$\prod_{i=k}^{k+s-1} (A_i-B_iG_i) \approx (A_k-B_kG_{k+s-1})(A_{k+1}-B_{k+1}G_{k+s-2})\cdots(A_{k+s-1}-B_{k+s-1}G_k)$$

Define then:

$$\varepsilon = \inf(\varepsilon_1,\varepsilon_2)$$

It results from what has preceded that if condition C_2 is true then there exists an integer r' and a real $q<1$ so that:

$$||\theta_i-\theta_{i-1}|| < \varepsilon \ (k \leq i \leq k+r') \Longrightarrow || \prod_{i=k}^{k+r'-1} (A_i-B_iL_i)|| < q < 1 \tag{D.3}$$

From property P_3 of the identifier, there exists a positive real R_1 so that:

$$||\phi_{t-1}|| > R_1 \Longrightarrow ||\theta_t-\theta_{t-1}|| < \varepsilon \tag{D.4}$$

According to relations (C.17) and (C.18), if we choose $R_1 \gg M_{\mathscr{E}}$ we can roughly write:

$$\Phi_{t+1} \simeq \mathscr{F}_t \ \Phi_t \qquad \text{where} \qquad \Phi_t = \begin{bmatrix} \hat{x}_t \\ \phi_{t-2} \end{bmatrix} \tag{D.5}$$

Let us show that $||\phi_t||$ cannot stay always larger than R_1. Let us give a proof by contradiction and assume that there exists a time t_1 so that:

$$t \geq t_1 \Longrightarrow ||\phi_t|| > R_1 \quad (\text{with } ||\phi_{t_1-1}|| \leq R_1) \tag{D.6}$$

According to relations (D.3) and (D.4):

$$|| \prod_{i=k}^{k+r'-1} (A_i-B_iL_i)|| < q < 1 \quad \text{for } k > t_1 \tag{D.7}$$

The matrix $F_t = A_t - B_t L_t$ is therefore uniformly exponentially stable for $t > t_1$, which entails (proof in Appendix B of Ref. 5) that the matrix $\mathcal{F}_t$ of relation (D.5) is uniformly exponentially stable.

According to (D.5), the vector Φ_t is uniformly and exponentially projected towards 0, as consequently ϕ_t. There therefore exists an integer t_2 (that does not depend on t_1 but on the constraint r') so that:

$$||\phi_{t_1}|| > R_1 \Longrightarrow ||\phi_{t_1+t_2}|| < R_1 \tag{D.8}$$

Relation (D.8) contradicts our starting assumption and therefore $||\phi_t||$ cannot always stay larger than R_1.

In fact, relation (D.8) shows more than that. It shows that $||\phi_t||$ cannot stay more than t_2 consecutive times larger than R_1. Now taking into account that $||\phi_t||$ cannot diverge faster than exponentially (relation (C.8)), this demonstrates that $\{\phi_t\}$ stays bounded.

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